Formelark -Mas 160-2014

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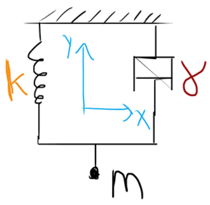


$$F_{total} = F_S + F_D + F_E$$

$$m \cdot y'' = -k \cdot y - \gamma \cdot y' + F_E$$

$$my'' + \gamma y' + ky = F_E$$

Diffrensialligninger

Partikulær løsning

Wronski metoden

$$|W| = \det\begin{pmatrix} y_1 & y_2 & \cdots & y_n \\ y'_1 & y'_2 & \cdots & y'_n \\ \cdots & \cdots & \cdots & \cdots \\ y''_1 & y''_2 & \cdots & y''_n \end{pmatrix}$$

Ved 2x2 snarvei:

$$y_p = y_1 \int \frac{-y_2 \cdot f(t)}{|W|}\,dt + y_2 \int \frac{y_1 \cdot f(t)}{|W|}\,dt$$

$$\boxed{DL = f(t)}$$

Ellers Cramers rule:

$$|W_1(b)| = \begin{vmatrix} 0 & y_2 & y_3 \\ 0 & y'_2 & y'_3 \\ f(t) & y''_2 & y''_3 \end{vmatrix},\ |W_2(b)| = \begin{vmatrix} y_1 & 0 & y_3 \\ y'_1 & 0 & y'_3 \\ y''_1 & f(t) & y''_3 \end{vmatrix},\ |W_3(b)| = \begin{vmatrix} y_1 & y_2 & 0 \\ y'_1 & y'_2 & 0 \\ y''_1 & y''_2 & f(t) \end{vmatrix}$$

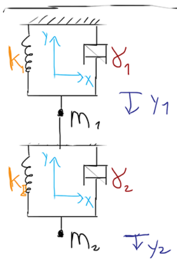
$$u'_1 = \frac{W_1(b)}{|W|},\ u'_2 = \frac{W_2(b)}{|W|},\ u'_3 = \frac{W_3(b)}{|W|}$$

$y_p = y_1 \int u'_n\,dt + \ldots + y_n \int u'_n\,dt \longrightarrow$ unødvendig og føre c

Kvadrat Ligning

$$ax^2 + bx + c = 0$$

$$x_{1,2} = -\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$



$F_{S_1} = -k_1 \cdot y_1$ $\quad F_{S_2} = -k_2(y_2 - y_1)$

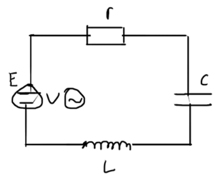
$F_{D_1} = -\gamma_1 \cdot y'_1$ $\quad F_{D_2} = -\gamma_2(y'_2 - y'_1)$

m_1: $F_1 = F_{S_1} + F_{D_1} - F_{S_2} - F_{D_2} + F_{E_1}$

$\Rightarrow m_1 y_1'' + k_1 y_1 + \gamma_1 y_1' - k_2(y_2 - y_1) - \gamma_2(y_2' - y_1') = F_{E_1}$

m_2: $F_2 = F_{S_2} + F_{D_2} + F_{E_2}$

$\Rightarrow m_2 y_2'' + k_2(y_2 - y_1) + \gamma(y_2' - y_1') = F_{E_2}$



$\Delta V_r = -I \cdot r$

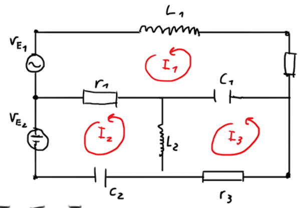
$\Delta V_C = -\int I\,dt \cdot C^{-1}$

$\Delta V_L = -I' \cdot L$

$\Delta V_r + \Delta V_C + \Delta V_L + \Delta V_E = 0$

$\Downarrow$ ganger m/-1 og deriverer

$LI'' + r \cdot I' + C^{-1} \cdot I = V_E'$



Krets 1 (I_1):

r_1: $\Delta V_{r_1} = -r_1(I_1 - I_2)$ $\quad r_2$: $\Delta V_{r_2} = -r_2 \cdot I_1$

C_1: $\Delta V_{C_1} = -C_1^{-1}(\int I_1 dt - \int I_3 dt)$ $\quad L_1$: $\Delta V_{L_1} = -L_1 \cdot I_1$

Ekstern: V_{E_1} $\quad = \sum = 0$

$\Rightarrow L_1 \cdot I_1'' + (r_1 + r_2) I_1' - r_1 I_2' + \frac{1}{C_1} I_1 - \frac{1}{C_1} I_3 = V_{E_1}'$

Transformasjon

speiling om origo $\mathbb{R}^3 \to \mathbb{R}^3$

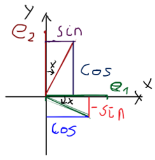
$$B = \left(T\begin{pmatrix}1\\0\\0\end{pmatrix} T\begin{pmatrix}0\\1\\0\end{pmatrix} T\begin{pmatrix}0\\0\\1\end{pmatrix} \right) = \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

Standardmatrise til ToS → Gang standardmatrise til T med standard til S v

SoT→Gang standardmatrise til S med standard til T

$T: \mathbb{R}^2 \to \mathbb{R}^2$ $x°$ ↺

$e_1 = \begin{bmatrix}1\\0\end{bmatrix}$, $T(e_1) = \begin{bmatrix}\cos x\\ -\sin x\end{bmatrix}$ $\quad e_2 = \begin{bmatrix}0\\1\end{bmatrix}$, $T(e_2) = \begin{bmatrix}\sin x\\ \cos x\end{bmatrix}$



$C = [T(e_1)\ T(e_2)] = \begin{bmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{bmatrix}$

$T: \mathbb{R}^3 \to \mathbb{R}^2$

$T: \begin{bmatrix}a\\b\\c\end{bmatrix} = \begin{bmatrix}a - 3b\\ b - 5c\end{bmatrix}$

$T_1 \begin{bmatrix}1\\0\\0\end{bmatrix} \xrightarrow[\text{resten}=0]{a=1} \begin{bmatrix}1-3\cdot 0\\ 0-5\cdot 0\end{bmatrix} = \begin{bmatrix}1\\0\end{bmatrix}$

$T_2 \begin{bmatrix}0\\1\\0\end{bmatrix} = \begin{bmatrix}0-3\cdot 1\\ 1+5\cdot 0\end{bmatrix} = \begin{bmatrix}-3\\1\end{bmatrix}$, $T_3 \begin{bmatrix}0\\0\\1\end{bmatrix} = \begin{bmatrix}0-3\cdot 0\\ 0-5\cdot 1\end{bmatrix} = \begin{bmatrix}0\\-5\end{bmatrix}$

$D = [T_1\ T_2\ T_3] = \begin{bmatrix}1 & -3 & 0\\ 0 & 1 & -5\end{bmatrix}$

Fouier-rekker

Cos-rekke:

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cdot \cos\left(n \cdot \frac{\pi}{L} \cdot x\right)$$

der

$$a_n = \frac{2}{L} \int_0^L f(x) \cdot \cos\left(n \cdot \frac{\pi}{L} x\right) dx$$

Sin-rekke:

$$f(x) = \sum_{n=1}^{\infty} b_n \cdot \sin\left(n \cdot \frac{\pi}{L} x\right)$$

der

$$b_n = \frac{2}{L} \int_0^L f(x) \cdot \sin\left(n \frac{\pi}{L} x\right) dx$$

Omregning Cos($n\pi$)

$\cos(n\pi) = (-1)^n$

$\cos(n\pi - 2\pi) = \cos(n\pi) = (-1)^n$

$\cos(n\pi - 3\pi) = (-1)^{n-3} = (-1)^n(-1)^{-3}$

$\cos\left(\left(n + \frac{1}{2}\right)\pi\right) = 0$

Omregning Sin($n\pi$)

$\sin(n\pi) = 0$

$\sin(n\pi - 3\pi) = 0$

$\sin\left(\left(n + \frac{1}{2}\right)\pi\right) = (-1)^n$

Delbrøksoppspalting

Eksempel:

Starter med uttrykket: $\dfrac{3s^2 + s + 8}{s^3 + 4s}$

Former om nevner: $\dfrac{3s^2 + s + 8}{s(s^2 + 4)}$

Splitter opp: $\dfrac{3s^2 + s + 8}{s(s^2 + 4)} = \dfrac{A}{s} + \dfrac{Bs + C}{(s^2 + 4)}$

Ganger alle ledd med fellesnevner, $s(s^2 + 4)$, og former om uttrykket til dette: $3s^2 + s + 8 = s^2\underbrace{(A + B)}_{=3} + \underbrace{Cs}_{=1} + \underbrace{4A}_{=8}$;

Setter opp matrise;

$$\begin{bmatrix} 1 & 1 & 0 & 3 \\ 0 & 0 & 1 & 1 \\ 4 & 0 & 0 & 8 \end{bmatrix} \Rightarrow \begin{matrix} A = 2 \\ B = 1 \\ C = 1 \end{matrix} \Rightarrow \frac{3s^2 + s + 8}{s(s^2 + 4)} = \frac{2}{s} + \frac{s + 1}{(s^2 + 4)}$$

Tabel for omforming av nevner til delbrøkoppspalting

Faktor i nevner:	Delbrøk:
$(s - \alpha)^n$	$\dfrac{A_n}{(s-\alpha)^n} + \dfrac{A_{n-1}}{(s-\alpha)^{n-1}} + \cdots + \dfrac{A_1}{(s-\alpha)}$
$(s + \alpha)^n + \omega$	$\dfrac{As + B}{(s+\alpha)^n + \omega}$
$s^2 + \omega^2$	$\dfrac{Bs + C}{s^2 + \omega^2}$
$(s^2 + \omega^2)^n$	$\dfrac{B_n s + C_n}{(s^2+\omega^2)^n} + \dfrac{B_{n-1}s + C_{n-1}}{(s^2+\omega^2)^{n-1}} + \cdots + \dfrac{B_1 s + C_1}{s^2 + \omega^2}$

Diagonaliseringen til A:

En matrise kan diagonaliseres om matrisen er kvadratisk og den har lineært uavhengige vektorer. *Vektorene er linjært uavhengige så lenge matrisen ikke har frie variabler.*

$A = P \cdot D \cdot P^{-1}$ $\qquad A^k = P \cdot D^k \cdot P^{-1}$

$e^A = P \cdot e^D \cdot P^{-1}$ $\qquad P = [\vec{V}_1, \vec{V}_2, \cdots, \vec{V}_k]$

A: Matrisen som skal skrives om.
D: Matrise med egenverdiene i diagonalen.
P: Egenvektorene som matrise.

1. Finn egenverdiene $\{\lambda_1, \lambda_2 \ldots \lambda_n\}$
2. Finn egenvektorene $\{\vec{V}_1, \vec{V}_2 \cdots \vec{V}_3\}$
3. Sett opp matrisene P, D og P^{-1}, og pass på at egenverdiene og egenvektorene stemmer med hverandre.

$$P = \begin{bmatrix} \vec{V}_1 & \vec{V}_2 & \cdots & \vec{V}_n \\ \Updownarrow & \Updownarrow & \cdots & \Updownarrow \end{bmatrix}$$

$$D = \begin{bmatrix} \lambda_1 & 0 & \cdots & 0 \\ 0 & \lambda_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \lambda_n \end{bmatrix}$$

Eigenverdi og enhetsvektor

$A = \begin{bmatrix} 4 & -1 & 6 \\ 2 & 1 & 6 \\ 2 & -1 & 8 \end{bmatrix}$ $\quad \boxed{|A - I\lambda| = 0}$

$$\begin{vmatrix} 4-\lambda & -1 & 6 \\ 2 & 1-\lambda & 6 \\ 2 & -1 & 8-\lambda \end{vmatrix} \begin{matrix} 4-\lambda & -1 \\ 2 & 1-\lambda \\ 2 & -1 \end{matrix} = -x^3 + 13x^2 - 40x + 36 = 0$$

$\lambda_1 = 2, \lambda_2 = 2, \lambda_3 = 9$

$\boxed{(A - \lambda I)\vec{x} = \vec{0}}$ eks med $\lambda_1 = 2$

$$\begin{bmatrix} (4-2) & -1 & 6 \\ 2 & (1-2) & 6 \\ 2 & -1 & (8-2) \end{bmatrix} \xrightarrow{\text{Rref}} \begin{bmatrix} 1 & -\frac{1}{2} & 3 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$x_1 = \frac{1}{2}x_2 - 3x_3$
$x_2 = \text{fri} = x_2$
$x_3 = \text{fri} = x_3$

$$\vec{x} = \begin{bmatrix} \frac{1}{2}x_2 - x_3 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} \frac{1}{2}x_2 \\ x_2 \\ 0 \end{bmatrix} + \begin{bmatrix} -3x_3 \\ 0 \\ x_3 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} \\ 1 \\ 0 \end{bmatrix} + \begin{bmatrix} -3 \\ 0 \\ 1 \end{bmatrix}$$

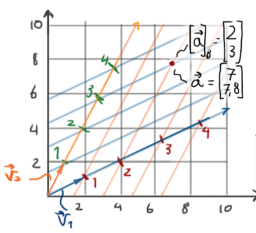
$\vec{V}_2 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}$ $\quad \vec{V}_3 = \begin{bmatrix} -3 \\ 0 \\ 1 \end{bmatrix}$

Koordinater med alternaltiv Basis

Mellom standar rom $\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$ og rom B

$B = \{\vec{V}_1, \vec{V}_2 \ldots \vec{V}_k\}$

$C = \begin{bmatrix} \uparrow & \uparrow & & \uparrow \\ \vec{V}_1 & \vec{V}_2 & \cdots & \vec{V}_k \\ \downarrow & \downarrow & & \downarrow \end{bmatrix}$ $\quad [\vec{a}]_B = \begin{bmatrix} C_1 \\ C_2 \\ \vdots \\ C_k \end{bmatrix}$



$[\vec{a}]_B = \begin{bmatrix}2\\3\end{bmatrix}$ $\quad \vec{a} = \begin{bmatrix}7\\8\end{bmatrix}$

$\boxed{C \cdot [\vec{a}]_B = \vec{a}}$

Regner på denne som en hvilken som helst $A\vec{x} = \vec{b}$ ligning.

eks: trappeform/invers

$\boxed{[\vec{a}]_B = C^{-1}\vec{a}}$

Fra rom B til Rom Q

$B = \left\{\begin{bmatrix}-9\\1\end{bmatrix}, \begin{bmatrix}-5\\-1\end{bmatrix}\right\}$ $\quad Q = \left\{\begin{bmatrix}1\\-4\end{bmatrix}, \begin{bmatrix}3\\-5\end{bmatrix}\right\}$

• Finn $P_{Q \leftarrow B}$

$[q_1\ q_2 \mid b_1\ b_2] \sim \left[\begin{array}{cc|cc} 1 & 3 & -9 & -5 \\ -4 & -5 & 1 & -1 \end{array}\right]$

$\sim \left[\begin{array}{cc|cc} 1 & 0 & 6 & 4 \\ 0 & 1 & -5 & 3 \end{array}\right] = [I_2 \mid P_{Q \leftarrow B}]$

$P_{Q \leftarrow B} = \begin{bmatrix} 6 & 4 \\ -5 & 3 \end{bmatrix}$

$\boxed{[\vec{X}]_Q = P_{Q \leftarrow B}[\vec{X}]_B}$ $\quad \boxed{(P_{Q \leftarrow B})^{-1} = P_{B \leftarrow Q}}$

Eksakt DL hjelpe fig.

② → H ← ③ ④

$N = H_y$ $\quad M = H_t$ starter her

① ↘ $H_{yt} = H_{ty}$ ↙ ①

1: Partiell deriver for test av eksakt.
2: Partiell antideriver for y del av utrykk
3: Partiell deriver på t for og løse C(t)
4: Partiell antideriver C'(t)
5: Løs ut y fra H=K